Homework #11 (10 points) - Show all work on the following problems:

Problem 1 (2 points): Find the exact magnetic field a distance z above the center of a square loop of side w, through which a steady current I flows. Verify that your answer reduces to the field of a dipole for $z >> w$.

Problem 2 (2 points): Find the magnetic dipole moment (magnitude and direction) of a spherical shell with radius R, carrying a uniform surface charge σ, and spinning around the z-axis with angular velocity ω.

Problem 3 (2 points): Use the formula $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$ to find the force between two perfect dipoles with magnitude m_1 and m_2, both lying on the z-axis, aligned in the $+z$ direction, and separated by a distance r.

Problem 4 (2 points): Find the magnetic field of an infinitely long cylinder with a uniform magnetization M parallel to its axis, for the region inside the cylinder (s < R) and the region outside the cylinder (s > R).

Problem 5 (2 points): Consider an infinitely long cylinder of radius R, with a permanent magnetization $\vec{M}(s) = ks\,\hat{z}$ that increases linearly with distance from the axis to the surface. Find the magnetic field inside and outside the cylinder using two methods:

5a (1 point): Locate all the bound surface and volume currents, and use Ampere's law for B (Eq. 5.57) to calculate the field inside and outside the cylinder.

5b (1 point): Use Ampere's law for H (Eq. 6.20), and then compute B from H and M.